Thinking about Learning and Teaching Sequences for the Addition and Subtraction of Fractions

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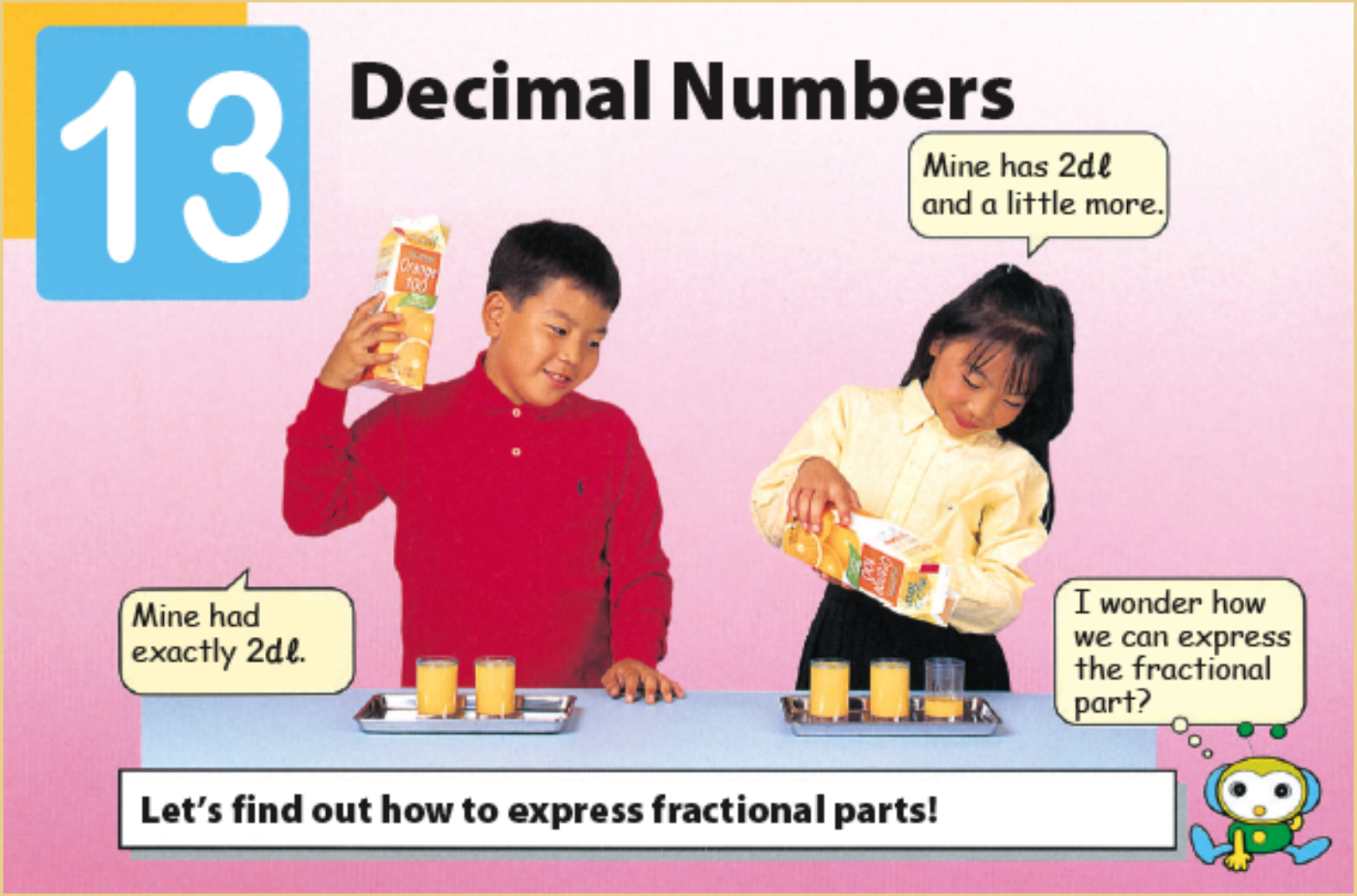
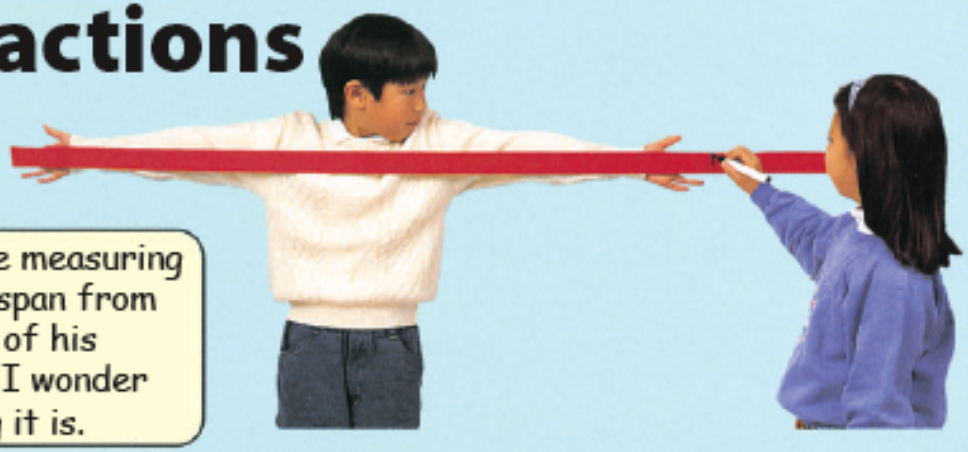
13 Decimal Numbers



3B p. 29

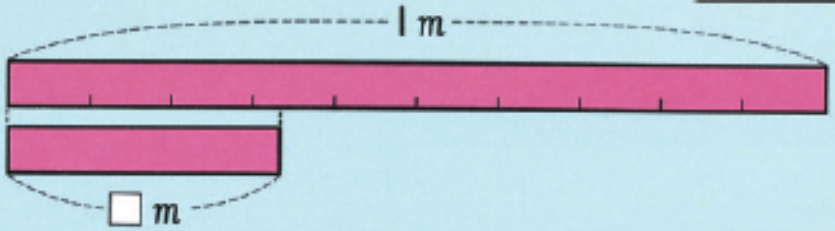
16 Fractions



They are measuring his arm span from the tips of his fingers. I wonder how long it is.

It is 1 m and a little more. We should use a decimal number.

Can you use a decimal number for this?



Let's think about how to express fractional parts!

- What similarities do you seen in these two contexts?
- What differences do you see in these two contexts?

- How are these problem contexts similar to or different from those that you are familiar with?

Similarities/Differences

- Both problems involve measurements.
  - One involves liquid measures while the other is about linear measurements.

Teaching Guide (p. 59)

- Children have learned to represent length and capacity in such manners as 9cm and 2mm, 3*l* and 6*dl*, or 1*dl* and a little more. Also, they have learned to represent sizes in such a manner as a half of something, or a half of a half of something through the experiences in their daily life. In this grade, based on these experiences, children are to learn to use decimal numbers and fractions to represent fractional parts…"

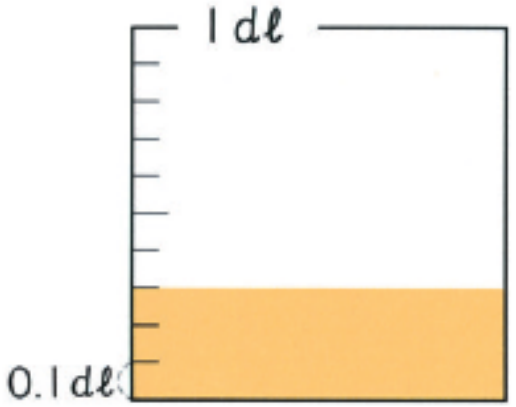
Similarities/Differences

- Both problems involve measurements.
  - One involves liquid measures while the other is about linear measurements.
- Both problems involve quantities greater than 1 unit
  - With decimals, the whole quantity is represented using the notation "2.3" but with fractions, textbook does not use a mixed number (or improper fraction) to express the whole length.

Similarities/Differences

- Decimals and fractions as collections of decimal/fraction units, that is, "0.3" is *thr*ee "0.1" units, and 3/4 is *thr*ee "1/4" units.

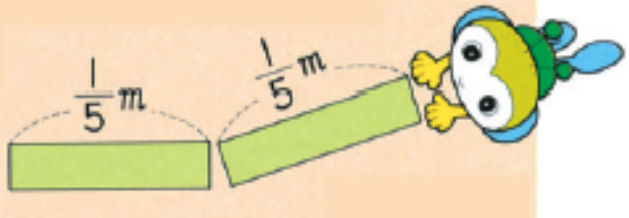
Each of 10 equal partitions of 1 dℓ is written as 0.1 dℓ. We say it as "Zero point one deciliters."



3 How many 0.1 dℓ are in the measuring cup above? How much is this?

p. 30

1 How long are 2, 3, and 4 pieces of $\frac{1}{5}$ m? Please express these as fractions on a number line.



p. 60

Meanings of fractions

Teaching Guide (p. 59) [2/3 as an example]

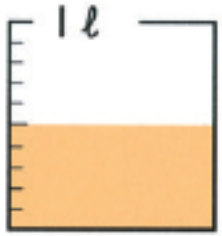
1. two of three equally divided parts
2. the quantity resulting from a measurement, such as 2/3 meter or 2/3 deciliter
3. twice the unit that is obtained when 1 is partitioned into three equal parts
4. the ratio of A to B, I.e., the relative size of A when B is considered as 1
5. the quotient of 2 ÷ 3

1 There are two juice bottles. The bigger one contains 0.5ℓ and the smaller one contains 0.3ℓ. How much juice is there altogether?

? Let's think about how to calculate 0.5+0.3!

1 How many 0.1's are in 0.5 and in 0.3?

0.5 ····· ☐ 0.1's
0.3 ····· ☐ 0.1's



0.5 + 0.3 = ☐

Answer: ☐ ℓ

5 There is $\frac{4}{5}\ell$ of juice.
If a girl drink $\frac{1}{5}\ell$ of juice, how much juice will be left?

? Let's think about how to calculate $\frac{4}{5} - \frac{1}{5}$!

1 How many $\frac{1}{5}\ell$ are in each $\frac{4}{5}\ell$ and in $\frac{1}{5}\ell$?

$\frac{4}{5}\ell \cdots \square \frac{1}{5}\ell$

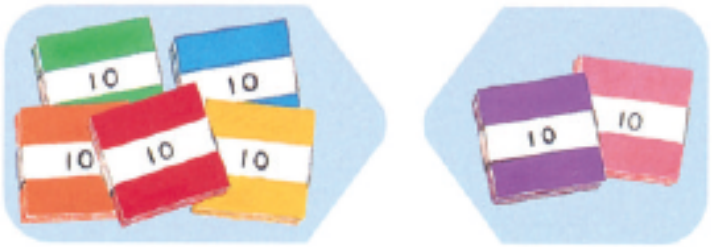
$\frac{1}{5}\ell \cdots \square \frac{1}{5}\ell$

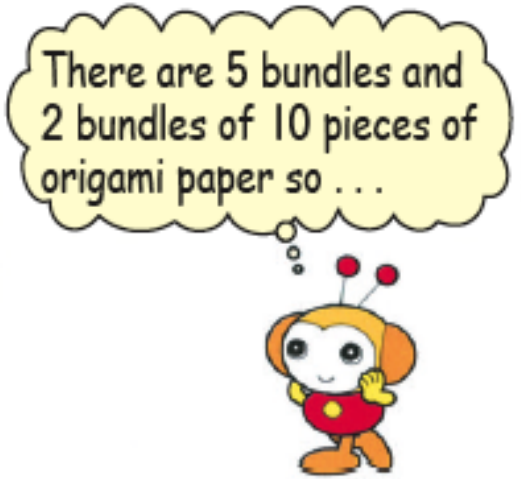
$\frac{4}{5} - \frac{1}{5} = \square$

Answer: $\square\ \ell$

How many 10's are there?

1 How many pieces of origami paper are there altogether?





$50 + 20 =$ ☐

Gr. 1 p. 93

Teaching Guide

p. 60 (Grade 3):

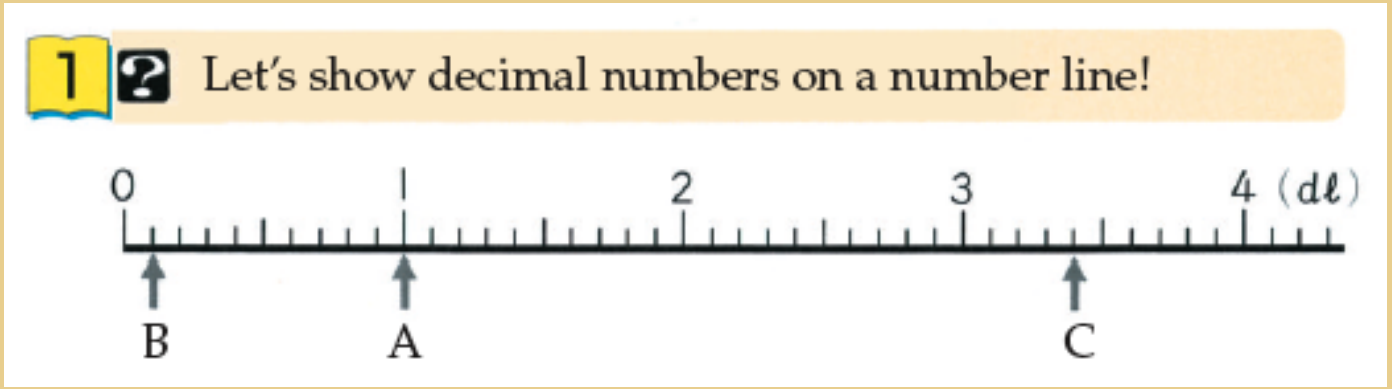
Calculations such as 1/5 + 2/5 can be thought of as the same as that of whole numbers, taking 1/5 as a unit.

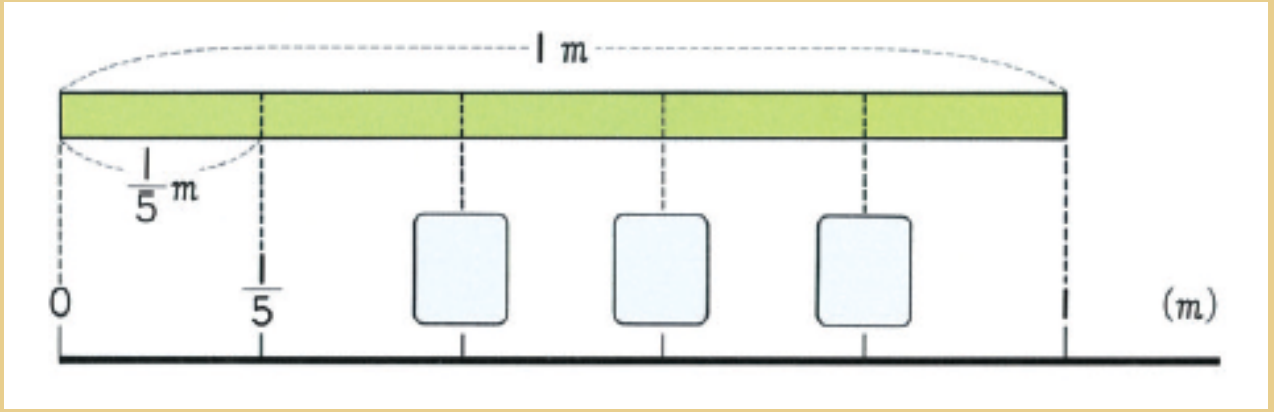
p. 69 (Grade 4):

… it is important to help children think of it as counting numbers of a fraction whose numerator is one so that they can see the similarity between addition and subtraction of whole numbers and fractions.

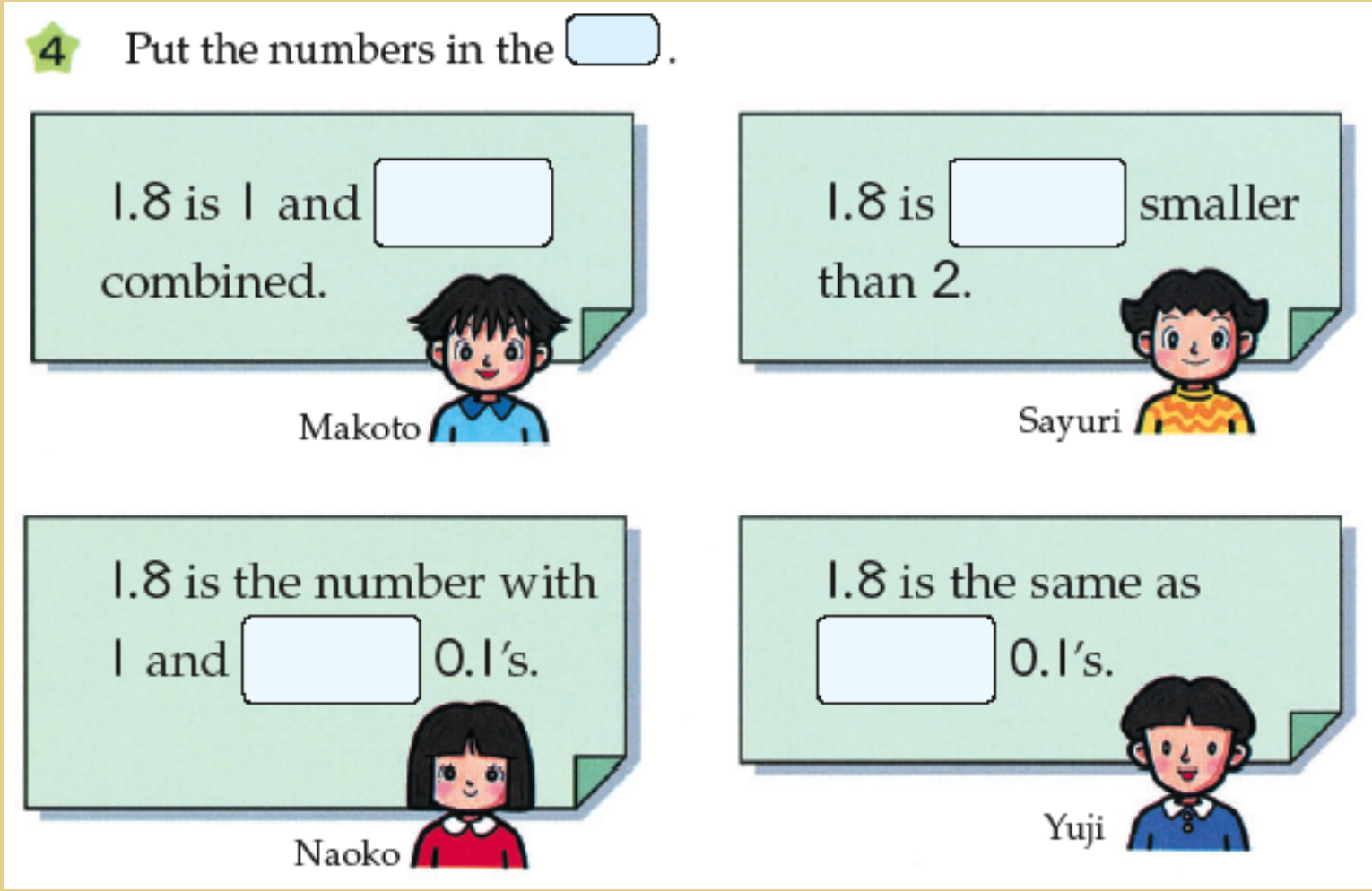
Decimals/Fractions as Numbers

- Representing decimals/fractions on a number line



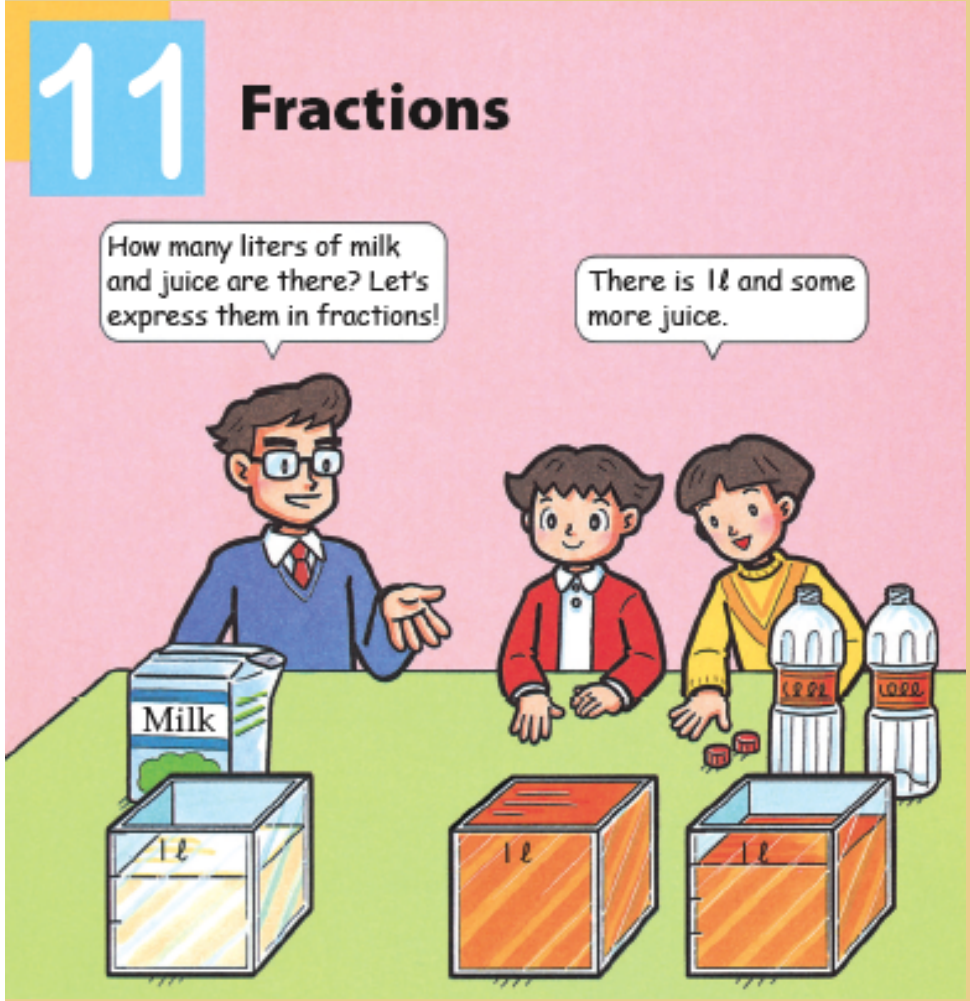


Describe the number 1.8 in as many different ways as you can. (3B, p. 33)



4B, pp. 38 - 39

11 Fractions



The quantity of milk

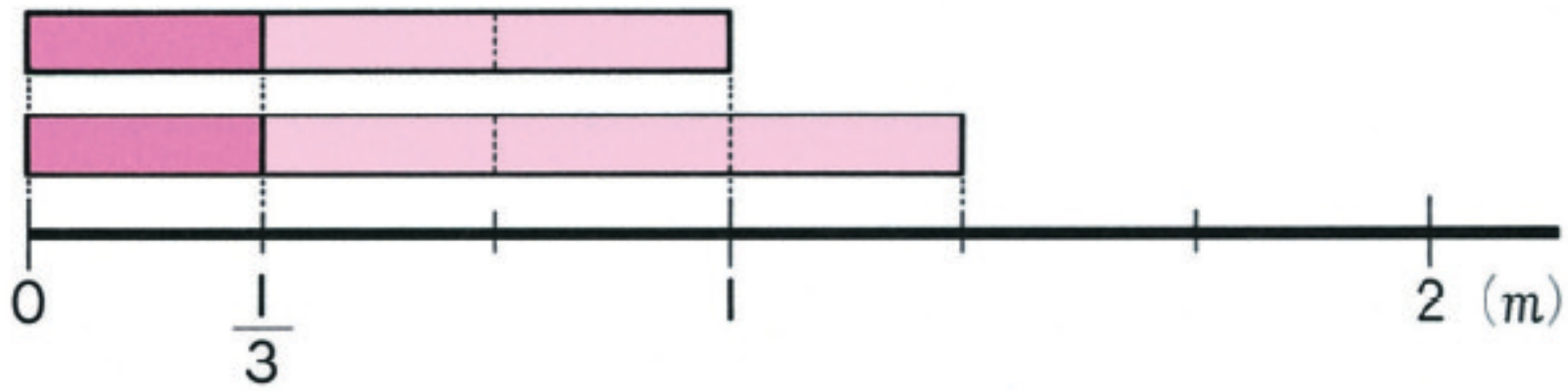
$\square \frac{1}{3}$ ℓ makes $\square$ ℓ

Let's think about various ways to express fractions!

Improper fractions, 4B, p. 41

Improper fractions

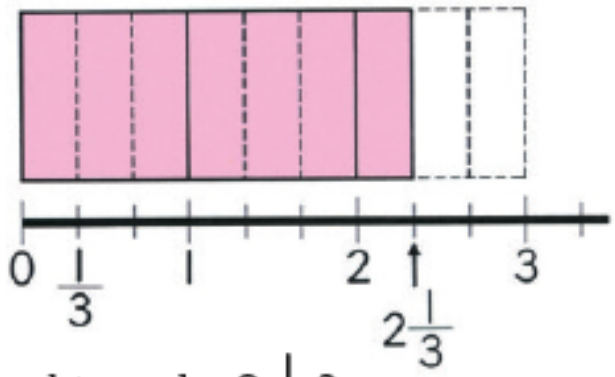
3 How many m are three $\frac{1}{3}m$? Four $\frac{1}{3}m$?



Mixed numbers and improper fractions (4B, p. 43)

Mixed numbers and improper fractions

5 Let's think about how to change $2\frac{1}{3}$ into an improper fraction!

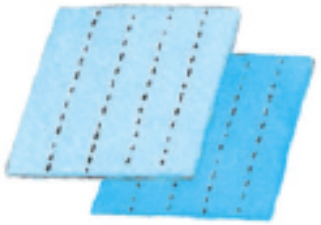


1 How many $\frac{1}{3}$'s do you need to make $2\frac{1}{3}$?

$3 \times 2 + 1 = \square$ $\qquad 2\frac{1}{3} = \frac{\square}{3}$

3 Addition and Subtraction of Fractions

1 Shiori used $\frac{3}{5}m^2$ of cardboard, and Kiyoshi used $\frac{4}{5}m^2$. How many m^2 of cardboard did they use altogether?

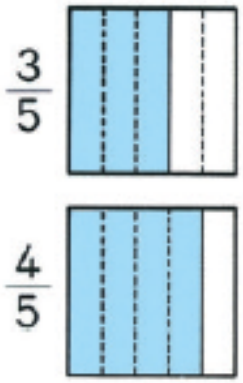


1 Write a math sentence.

Math Sentence ⇨ ☐

? Let's think about how to calculate!

2 How many $\frac{1}{5}$'s do you need to make $\frac{3}{5}$ and $\frac{4}{5}$?



$$\frac{3}{5}+\frac{4}{5}=\frac{\square}{5}$$

$$=\square\frac{\square}{5}$$

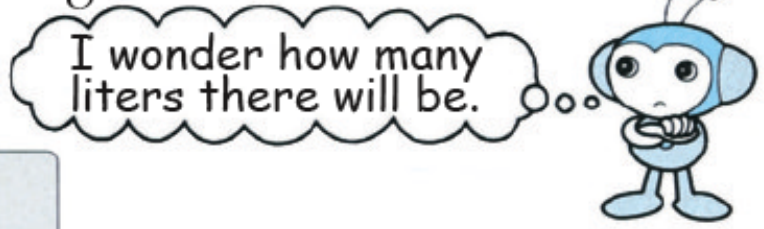
Answer: $\square\frac{\square}{5}m^2$

Addition of Decimal Numbers

(4B, p. 13)

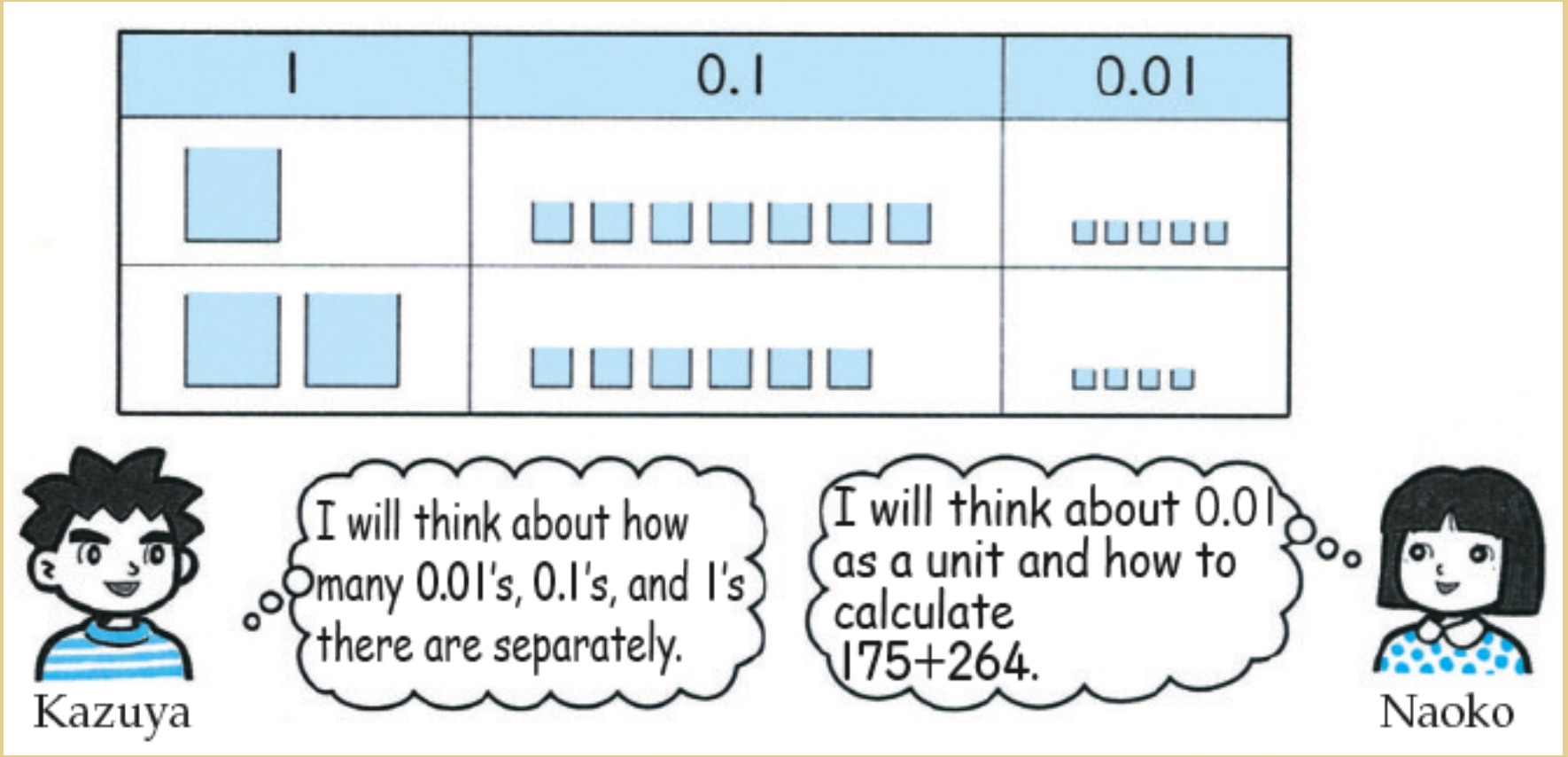
3 Addition and Subtraction of Decimal Numbers

1 The thermos has 1.75ℓ of water, and the tea kettle has 2.64ℓ. How many liters of water are there altogether?



Math Sentence

Let's think about how to calculate!


1
0.1
0.01
I will think about how many 0.01's, 0.1's, and 1's there are separately.
I will think about 0.01 as a unit and how to calculate 175+264.
Kazuya
Naoko

How to calculate 1.75+2.64

❶

$$\begin{array}{r} 1.75 \\ +\ 2.64 \\ \hline \end{array}$$

Write the numbers by lining up the place values.

❷

$$\begin{array}{r} 1.75 \\ +\ 2.64 \\ \hline 439 \end{array}$$

Just like whole number addition, start calculating from the right-most place.

❸

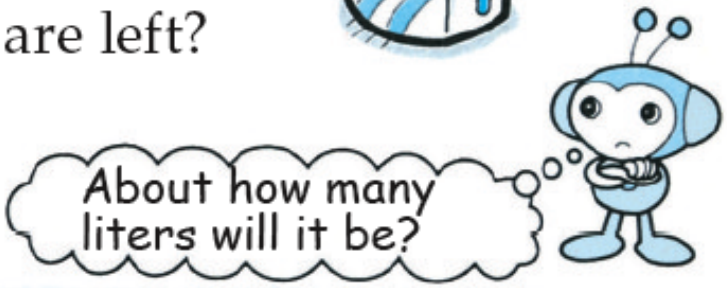
$$\begin{array}{r} 1.75 \\ +\ 2.64 \\ \hline 4.39 \end{array}$$

Write the decimal point right under-neath the other decimal points.

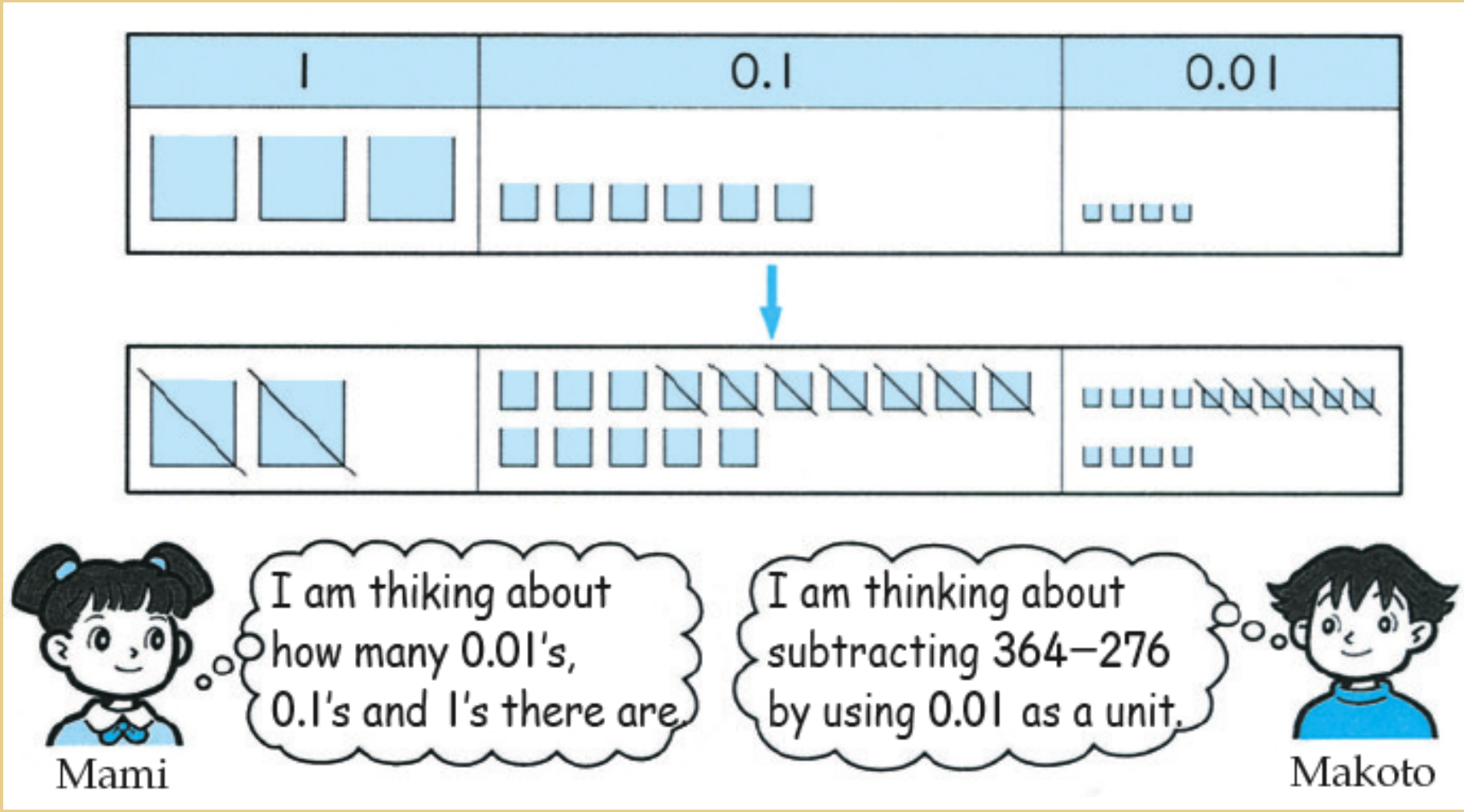
Subtraction of Decimal Numbers

4 Out of 3.64ℓ of water, 2.76ℓ were used.
How many liters of water are left?

Math Sentence



Let's think about how to calculate!


1
0.1
0.01
I am thiking about how many 0.01's, 0.1's and 1's there are.
Mami
I am thinking about subtracting 364−276 by using 0.01 as a unit.
Makoto

How to calculate 3.64−2.76

❶

$$\begin{array}{r} 3.64 \\ -\ 2.76 \\ \hline \end{array}$$

Write the numbers, lining up the place values.

❷

$$\begin{array}{r} 3.64 \\ -\ 2.76 \\ \hline 0.88 \end{array}$$

Just like whole number subtraction, start calculating from the right-most place.

❸

$$\begin{array}{r} 3.64 \\ -\ 2.76 \\ \hline 0.88 \end{array}$$

Write the decimal point right underneath the other decimal points and then write down a 0.

2 Equivalent Fractions

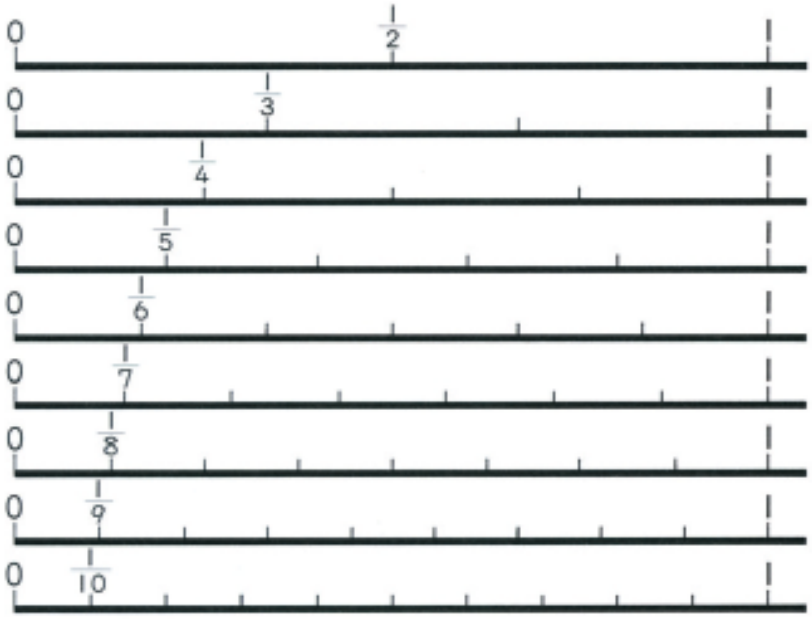
1 Let's express the colored parts below as fractions!

$\frac{1}{2}$, $\frac{2}{4}$, $\frac{5}{10}$ are equivalent fractions.

$\frac{1}{2} = \frac{2}{4} = \frac{5}{10}$

$\frac{1}{2}$ can be expressed as various fractions.

2 Let's find fractions that are equivalent using the number lines below!

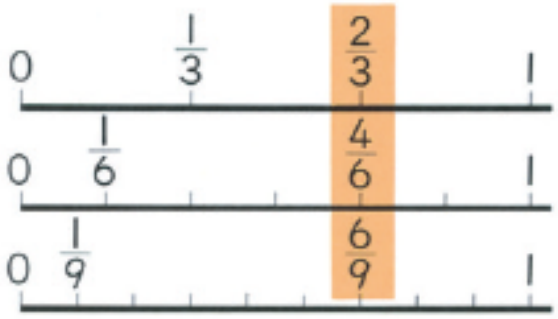


Equivalent Fractions

Grade 4: Become aware of their existence

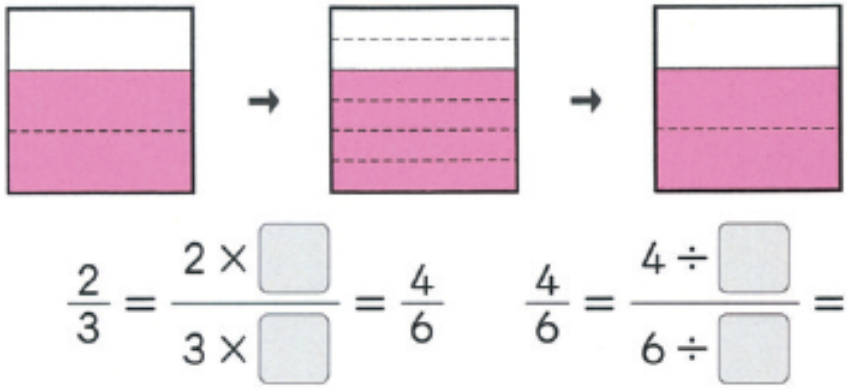
Equivalent fractions

1 $\frac{2}{3}$, $\frac{4}{6}$, and $\frac{6}{9}$ are equivalent fractions. Let's think about how to create equivalent fractions.



Equivalent Fractions

Grade 5: Learn how to create equivalent fractions



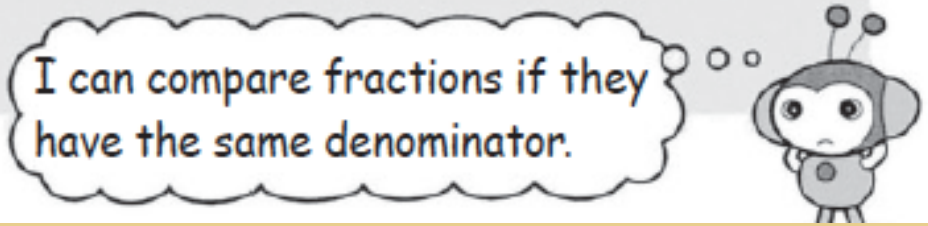
$$\frac{2}{3} = \frac{2 \times \square}{3 \times \square} = \frac{4}{6} \qquad \frac{4}{6} = \frac{4 \div \square}{6 \div \square} = \frac{2}{3}$$

$$\frac{2}{3} = \frac{2 \times \square}{3 \times \square} = \frac{6}{9} \qquad \frac{6}{9} = \frac{6 \div \square}{9 \div \square} = \frac{2}{3}$$

Comparing Fractions

3 Which fraction is greater, $\frac{3}{5}$ or $\frac{2}{3}$?

Let's think about how to compare fractions with different denominators!



1 Let's think about the following problems!

(1) There is $\frac{1}{3}$ ℓ of milk in a bottle and $\frac{1}{2}$ ℓ in a milk carton. How many ℓ of milk do we have altogether?

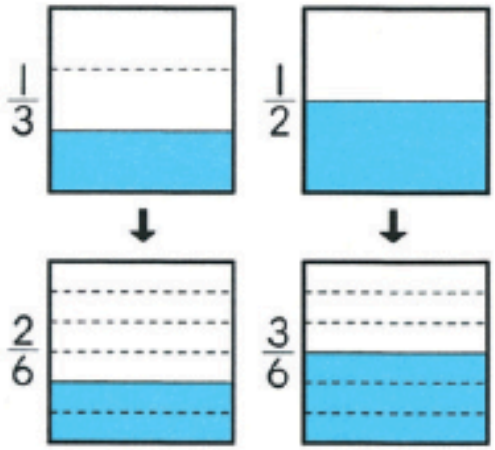
Math Sentence

? Let's think about how to calculate!

$$\frac{1}{3}+\frac{1}{2}=\frac{2}{6}+\frac{3}{6}$$

$$=$$

Answer: ℓ

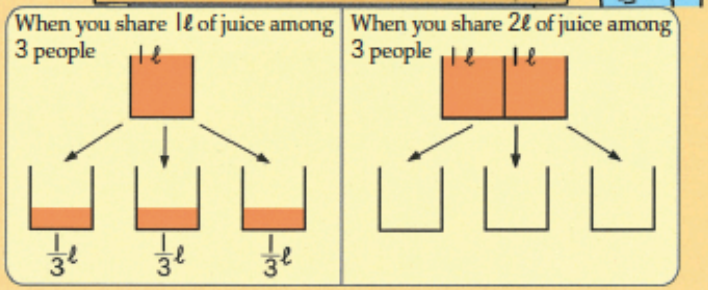


(2) We have $\frac{2}{3}$ ℓ of milk. We used $\frac{1}{2}$ ℓ to make a cake. How many ℓ of milk are left?

Math Sentence

Addition and subtraction of fractions with unlike denominators

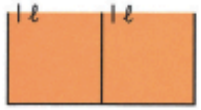
11 Fractions and Decimal Numbers



Let's think about the relationship between division and fractions!

1 Fractions and Division

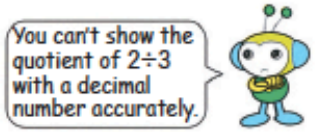
1 If you share 2ℓ of juice among 3 people, how many ℓ of juice will each person get?



$2 \div 3$

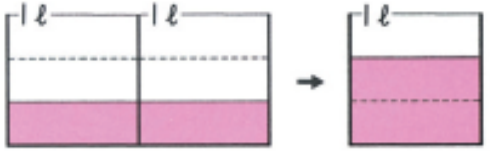
1 Please express the quotient of $2 \div 3$ as a decimal number.

$2 \div 3 = 0.666\ldots$, and the process does not stop.



? Let's think about another way to show the answer for division!

2 Look at the figure below and express the amount you get when you divide 2ℓ into 3 equal amounts using a fraction.



$2 \div 3 = \square\!\!\square\ (\ell)$

The quotient of $2 \div 3$ can be shown accurately by using a fraction.

3 When you share 4ℓ of juice among 3 people, how many ℓ of juice will each person get?

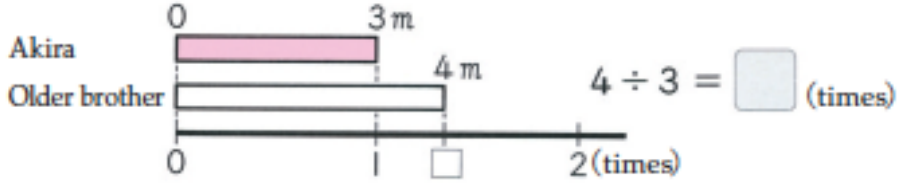
$4 \div 3 = \square\!\!\square = \square\ (\ell)$

! The quotient of the division of two whole numbers can be expressed as a fraction. $a \div b = \frac{a}{b}$

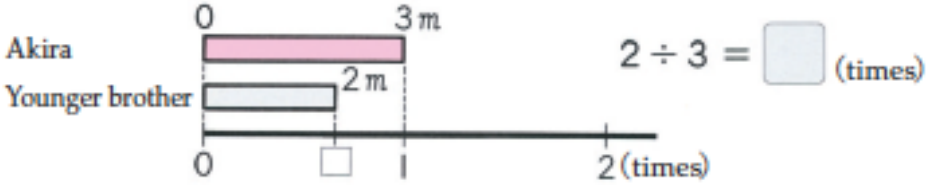
Times as much

2 Akira and his brothers did the long jump. Akira jumped 3m, his older brother jumped 4m, and his younger brother jumped 2m.
If you compare the other jumps to Akira's record, how many times as far are each of his brother's jumps as Akira's jump?

1 What kind of calculation do we need to do?



$4 \div 3 =$ ☐ (times)



$2 \div 3 =$ ☐ (times)

When you talk about how many times as many/much, we can use fractions like $1\frac{1}{3}$ times or $\frac{2}{3}$ times.
$\frac{2}{3}$ times as much represents the relationship that if 3m were 1, 2m would be $\frac{2}{3}$.

Two changes from 1989 to 2010

- ½ and ¼ are now introduced in Grade 2 through folding papers.
- Fractions > 1 in Grade 3, but expressed only as improper fractions.

Contrast between US and Japan

- Decimal numbers are introduced first.
- Initial contexts include numbers > 1.
- Calculations with decimals precede calculations with fractions
- Linear easurement and liquid measures (capacity/volume) are the primary contexts to discuss introductory ideas of fractions and decimals.
- Very limited use of area model.

Essential Ideas

- Fractions (and decimals) composed of units → bridging the fraction world and whole number world.
- Consistent use of models and problem situations.
  - linear (tape) models
  - area models
  - number line
  - measurement contexts
- Coherent sequence of ideas.